



Electric drives

Lecture 3 – D.C. Brush Motor

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MŠMT
MINISTERSTVO ŠKOLSTVÍ,
MLÁDEŽE A TĚLOVÝCHOVY



Projekt ESF CZ.1.07/2.2.00/28.0050
**Modernizace didaktických metod
a inovace výuky technických předmětů.**

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ



Advantages and disadvantages

Introduction

Construction

Principle

Induced voltage

Torque

Model

Start up

Breaking

Control

- Continuous speed control
- Accurate torque, velocity and position regulation
- Commutation
- Brushes
- Signification
- More exacting servicing

On account of presence of commutation and brushes are these motors replace by electronic commutator machines.



Usage in praxis

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- D.C brush motor has gigantic substitution in praxis
- Winding engines
- Rolling mills
- Heavy industry – in the applications with heavy masses, where is necessary continuous speed control

Construction

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Construction

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Induced voltage

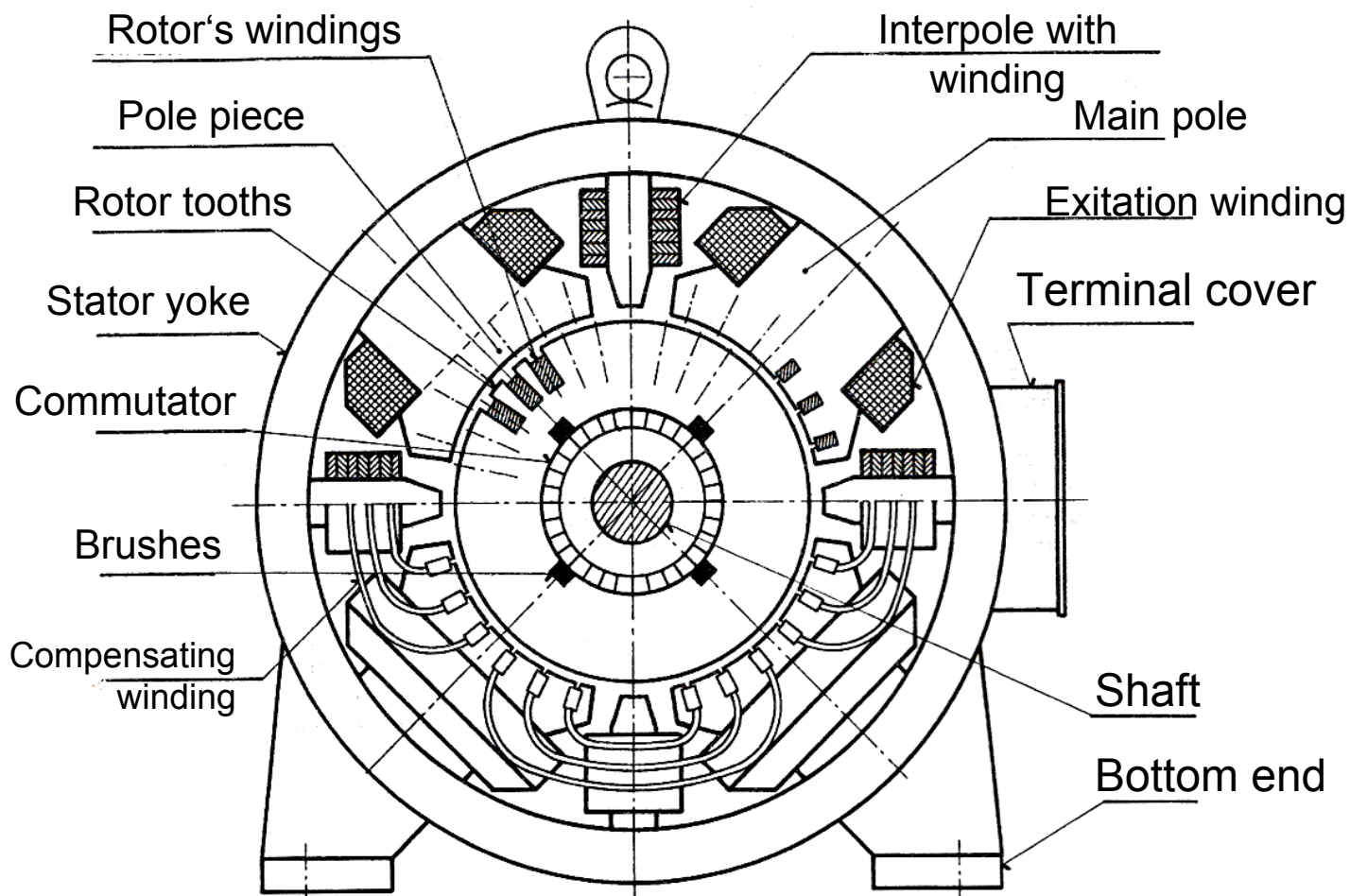
Torque

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Construction

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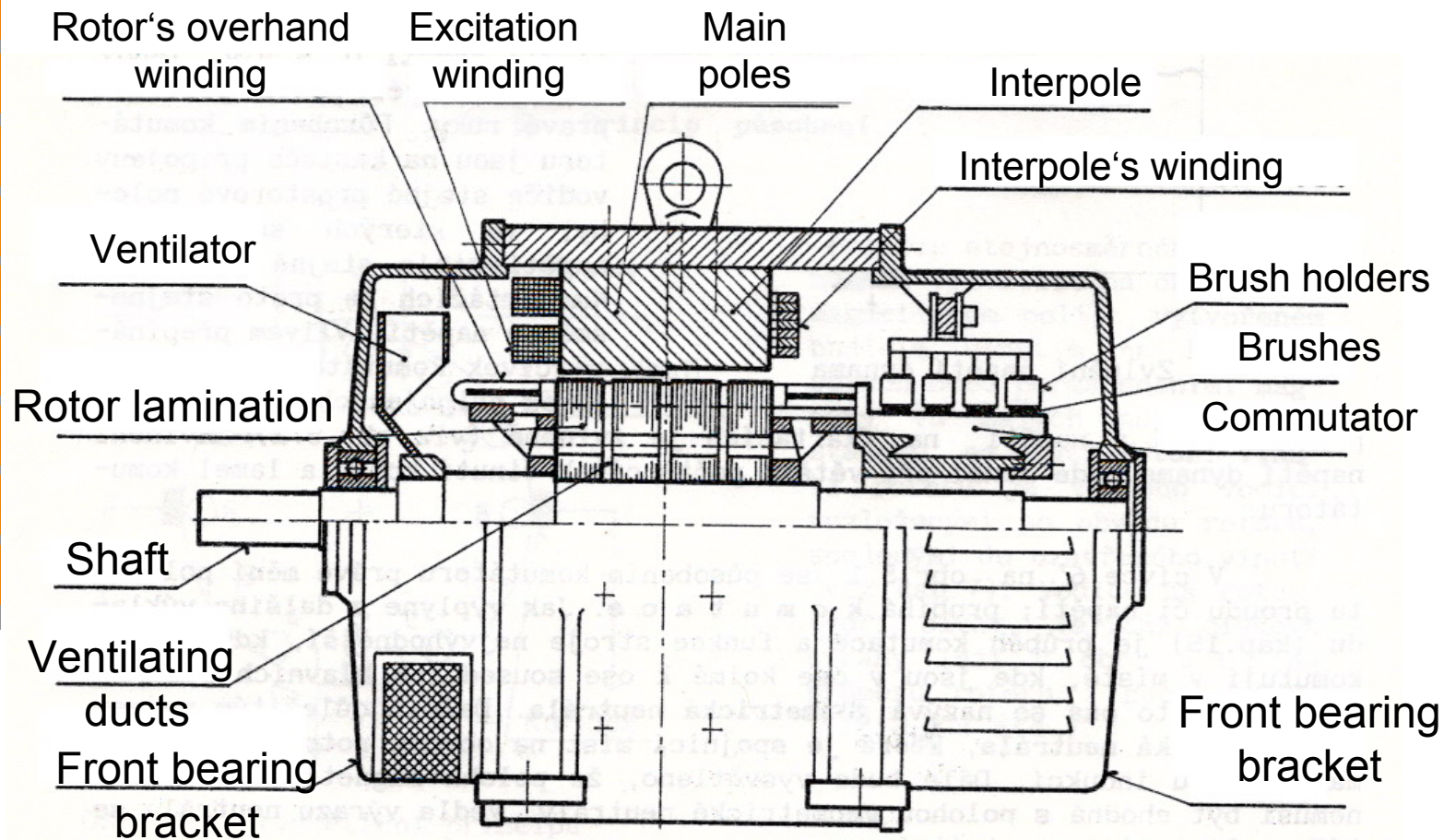
Torque

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Motor and dynamo principle

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Induced voltage

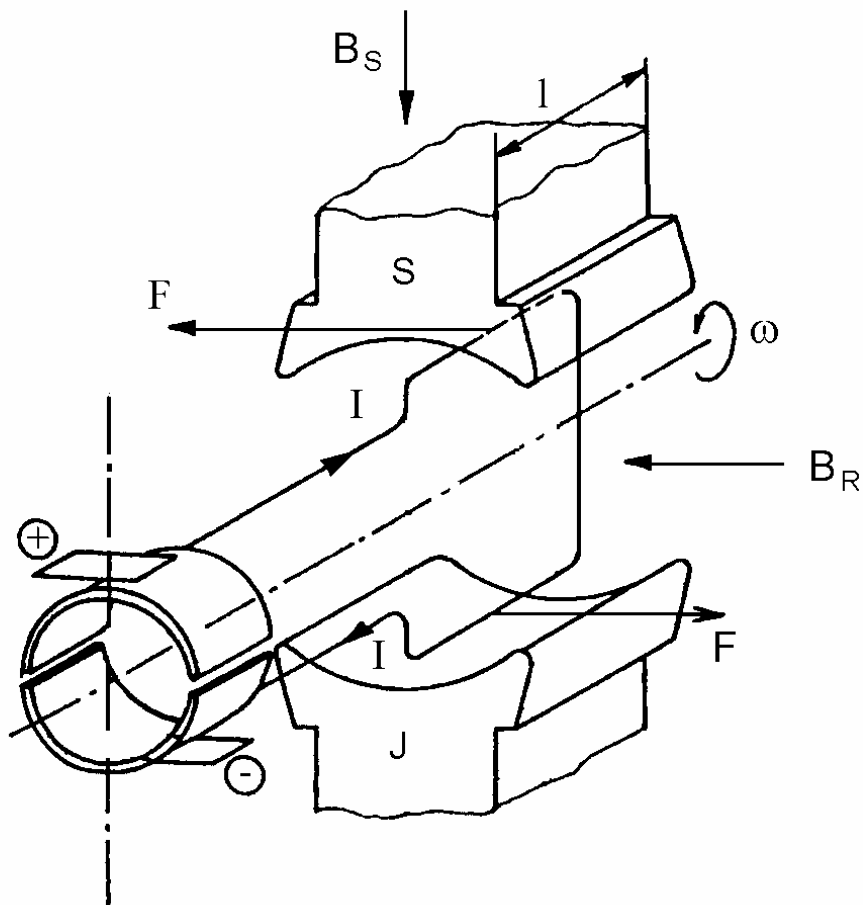
Torque

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Size of induced voltage

- Size of induced voltage D.S. motor is deduced from induced voltage i one wire

$$U_{iv} = B_x \cdot l_i \cdot v$$

Magnetic
inductance in the
place x on rotor
circumference

Active wire lenght

Wire velocity

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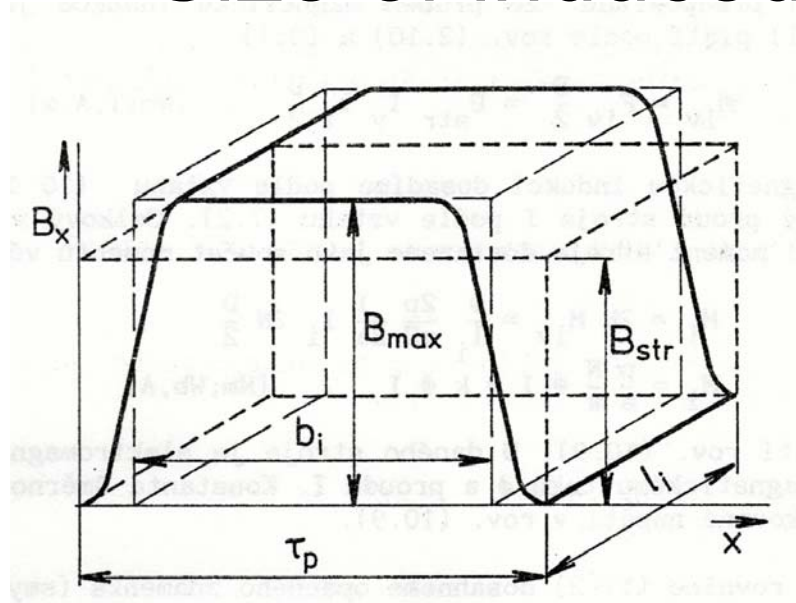
Model

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Size of induced voltage



$$\Phi = l_i \cdot \int_0^{\tau_p} B_x dx = B_{avg} \cdot l_i \cdot \tau_p = B_{mas} \cdot l_i \cdot b_i$$

$$B_{avg} = \frac{\Phi}{l_i \cdot \tau_p} = \frac{\Phi}{l_i} \frac{2p}{\pi \cdot D}$$

$$v = \omega \frac{D}{2}$$



Size of induced voltage

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- Machine has $2N$ wires connected to $2a$ parallel branches
- The whole machine voltage is done by multiplying one wire voltage by number of wires in one parallel branches

$$U_i = \frac{\Phi}{l_i} \frac{2p}{\pi D} \cdot l_i \cdot \omega_m \cdot \frac{D}{2} \cdot \frac{2N}{2a} = \frac{pN}{a\pi} \cdot \Phi \cdot \omega_m = k \cdot \Phi \cdot \omega_m$$



Electromagnetic torque

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- For size of electromagnetic torque M , which possesses on one wire in on rotor, we use equation:

$$M_{iw} = F_{iw} \cdot \frac{D}{2} = B_{avg} \cdot I_w \cdot l_i \cdot \frac{D}{2}$$

$$I_w = \frac{I}{2a}$$

$$B_{avg} = \frac{\Phi}{l_i} \cdot \frac{2p}{\pi D}$$

- The whole electromagnetic torque is sum of all wires $2N$



Electromagnetic torque

$$M_i = 2N \cdot M_{iw} = \frac{\Phi}{l_i} \cdot \frac{2p}{\pi D} \cdot \frac{I}{2a} \cdot l_i \cdot 2N \frac{D}{2} =$$

$$= \frac{p}{a} \cdot \frac{N}{\pi} \cdot \Phi \cdot I = k \cdot \Phi \cdot I$$

pole number

winding number

$$k = \frac{p}{a} \cdot \frac{N}{\pi}$$

branches number

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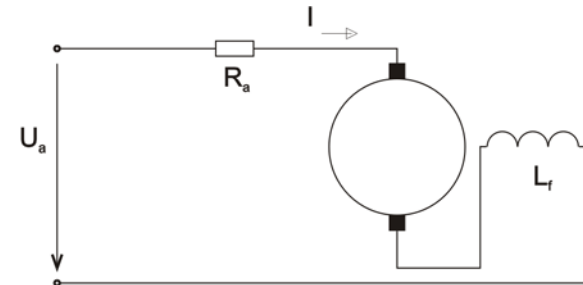
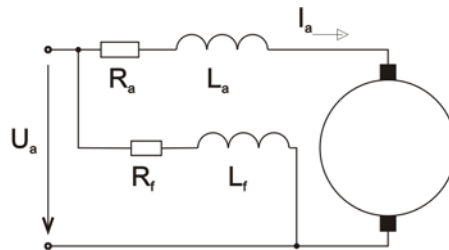
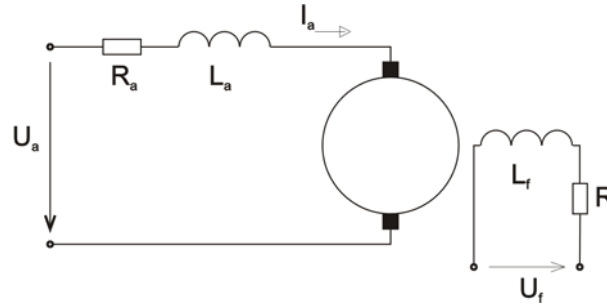
Start up

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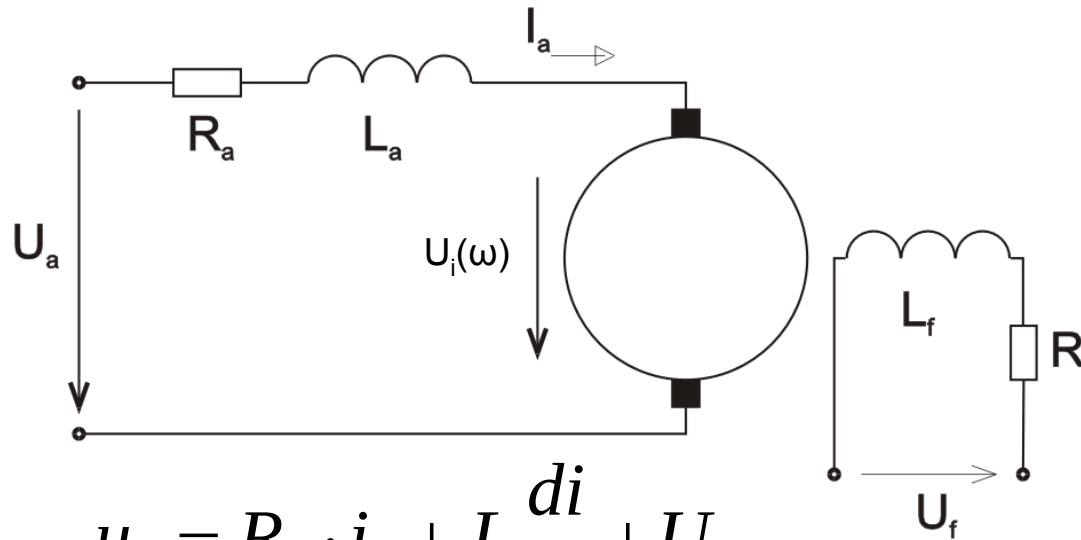
Types of D.C. motors

- D.C. brush motors with separate excitation
- D.C. brush motors with series excitation
- D.C. brush motors with parallel excitation



Mathematical description of D.C brush motor with external excitation

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$$u_a = R_a \cdot i_a + L \frac{di}{dt} + U_i$$

$$U_i = k \cdot \Phi \cdot \omega$$

$$M_i - M_z = J \frac{d\omega}{dt}$$

$$M_i = k \cdot \Phi \cdot i_a$$

Mathematical description of D.C brush motor with external excitation

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$$u_a = R_a \cdot i_a + L \frac{di_a}{dt} + c\Phi \omega$$

$$L \frac{di_a}{dt} = u_a - R_a i_a - c\Phi \omega$$

$$\frac{di_a}{dt} = \frac{1}{L_a} [u_a - R_a i_a - c\Phi \omega]$$

$$i_a = \frac{1}{L_a} \int [u_a - R_a i_a - c\Phi \omega]$$

Mathematical model of D.C brush motor with external excitation

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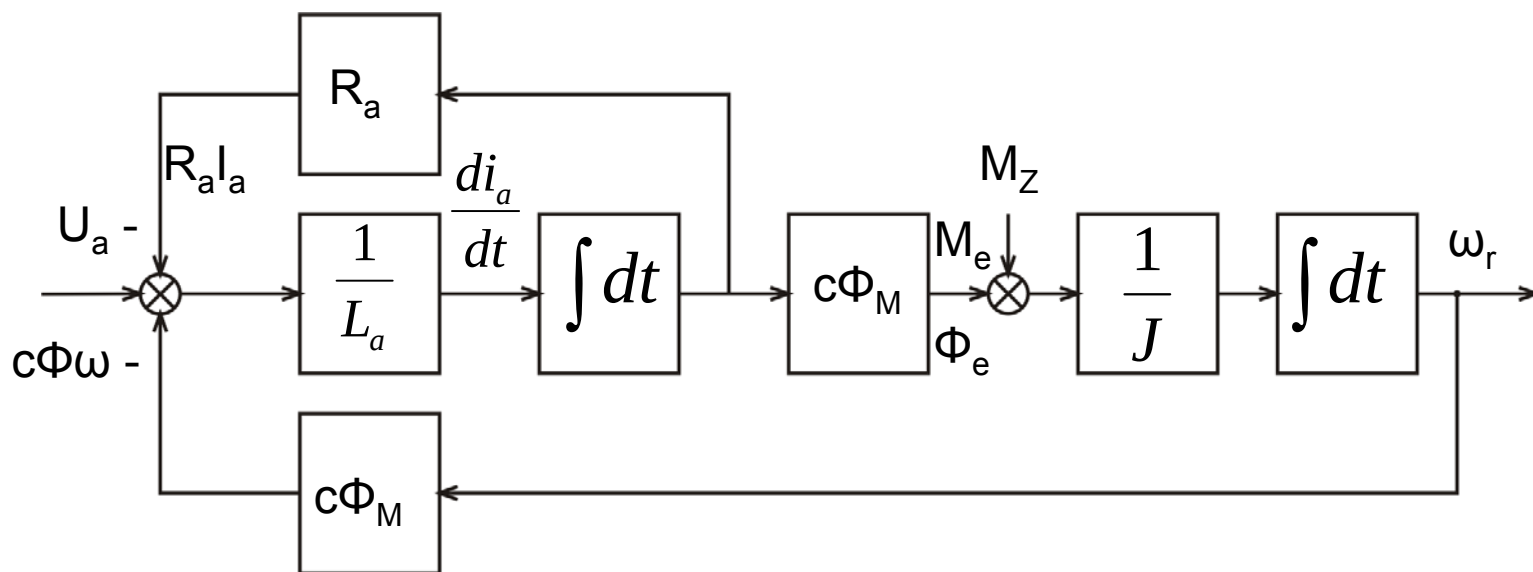
Torque

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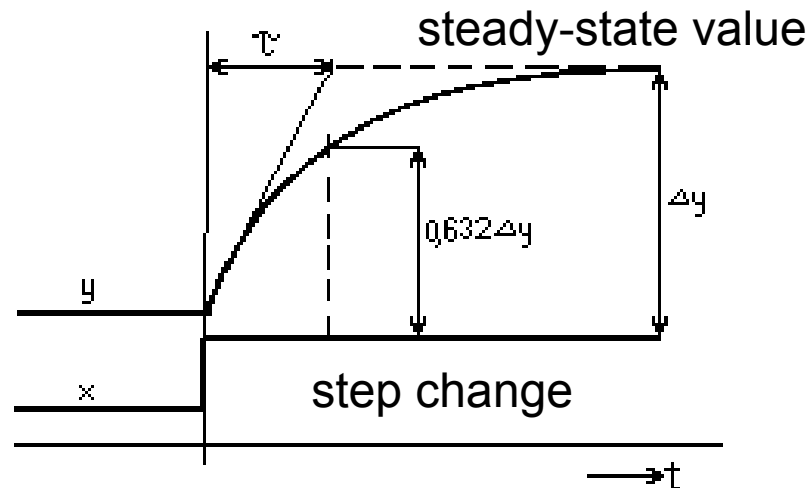
Breaking

Control



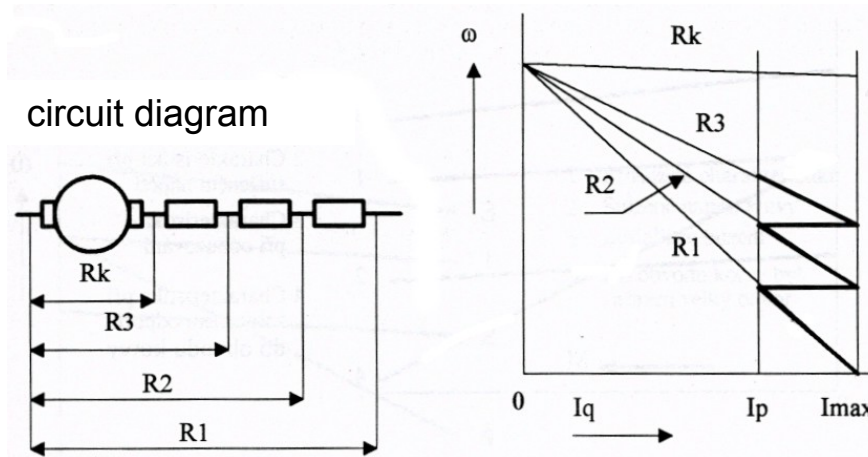
How to get the parameters for model?

- J – from D'alambert's principle, from the free slow-down stop
- R_a – by measuring from armature circuit
- L_a – from voltage step change and from I – curve



D.C. brush motor starting

- Resistance starting



Starting resistance proposal

$$R_1 = \frac{U}{I_{MAX}}$$

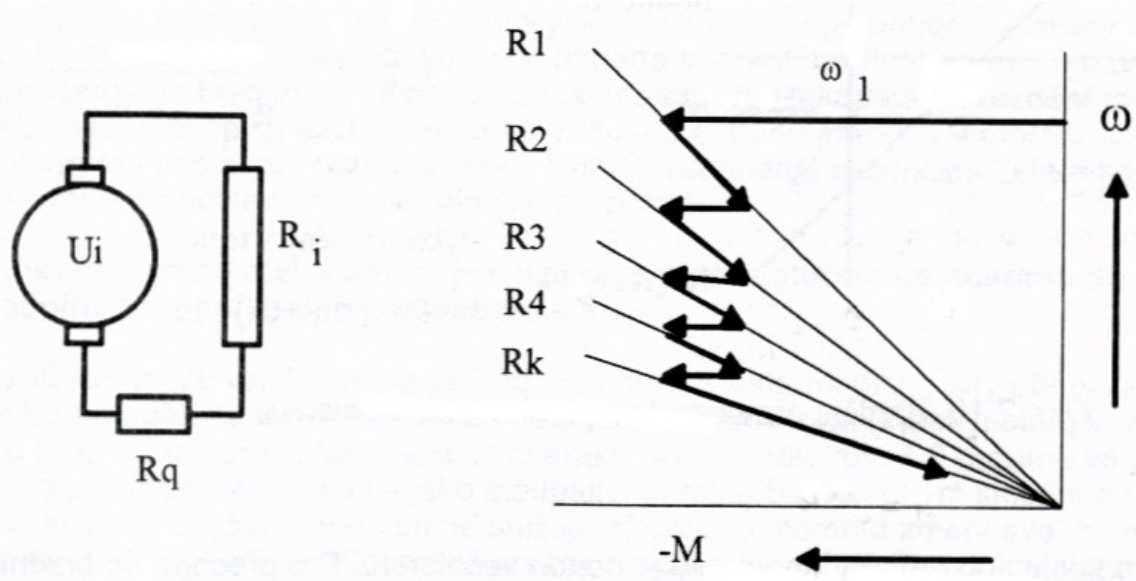
$$\frac{R_1}{R_2} = \frac{R_2}{R_3} = \frac{R_3}{R_q} = \dots = \frac{R_m}{R_q} = \frac{I_{max}}{I_p} = g$$

$$R_1 = g^m \cdot R_q \rightarrow R_2 = g^{m-1} \cdot R_q \rightarrow R_3 = g^{m-2} R_q$$

$$R_1 = \frac{U}{I_{max}} = g^m \cdot R_q \rightarrow m = \frac{\log \frac{U_q}{R_q I_{max}}}{\log g}$$

D.C. brush motor breaking

- Breaking to the armature resistor



$$0 = (R_a + R_B) \cdot I_a + k\Phi \omega \rightarrow \omega = \frac{R_a + R_B}{k\Phi} I_a$$

$$\omega = \frac{R_a + R_B}{(c\Phi)^2} \cdot M$$

D.C. brush motor breaking

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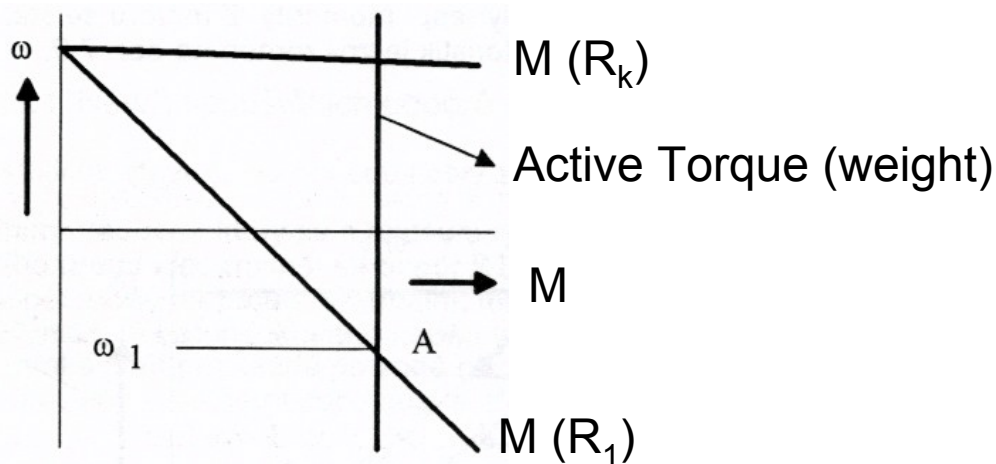
Model

Start up

Breaking

Control

- counter-current braking at active torque

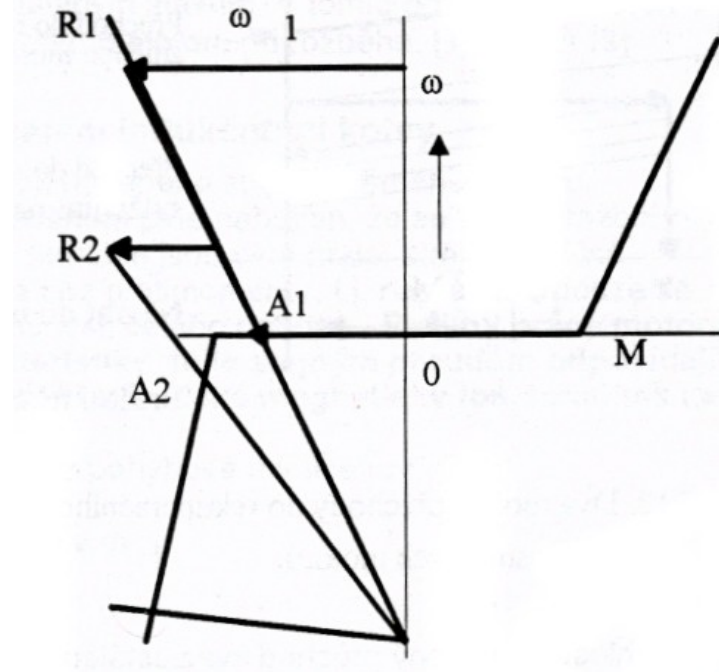


$$I = \frac{U + U_i}{R_a + R_1}$$



D.C. brush motor breaking

- counter-current braking at passive torque



R1 – mechanical characteristic

A1 – steady state after braking, motor stands still

A2 – drive run in the motor mode with reversal

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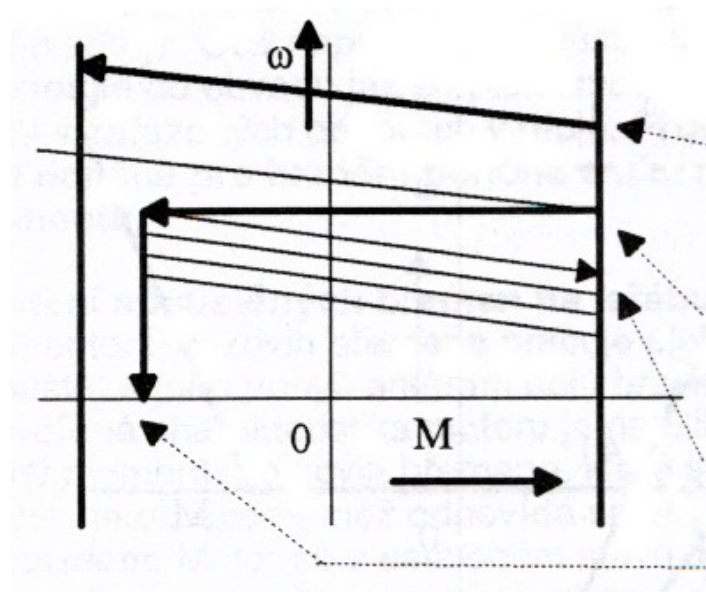
Start up

Breaking

Control

D.C. brush motor breaking

- regenerative braking



Crossing to the regenerative braking by the changing of the torque and velocity increasing

Crossing to the regenerative braking by the decreasing of the supplying voltage

Return to the motor operation

Recuperation breaking by the voltage decreasing

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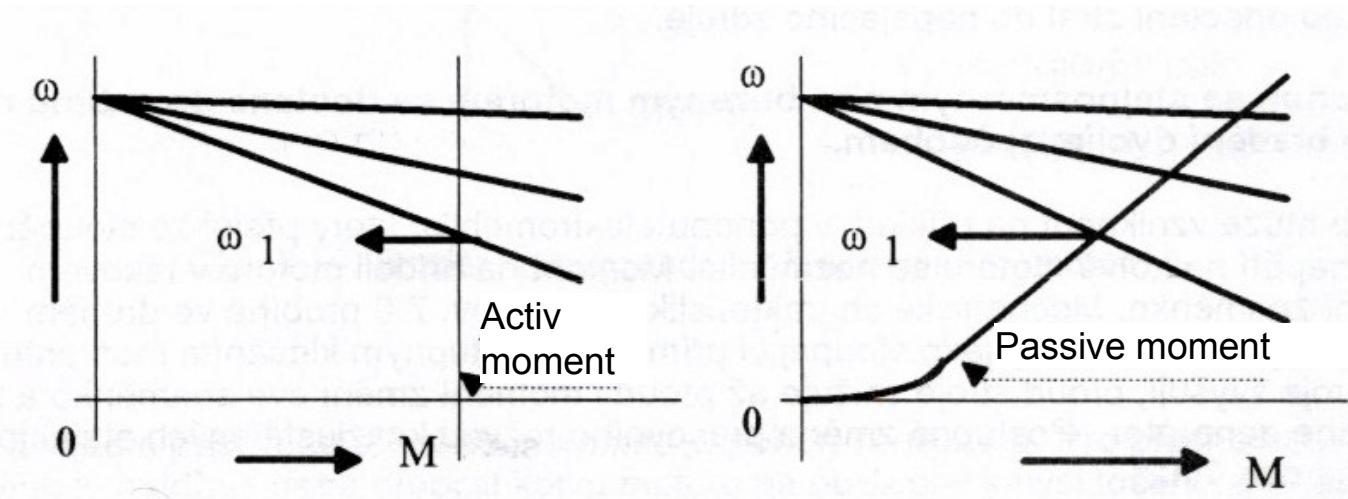
Start up

Breaking

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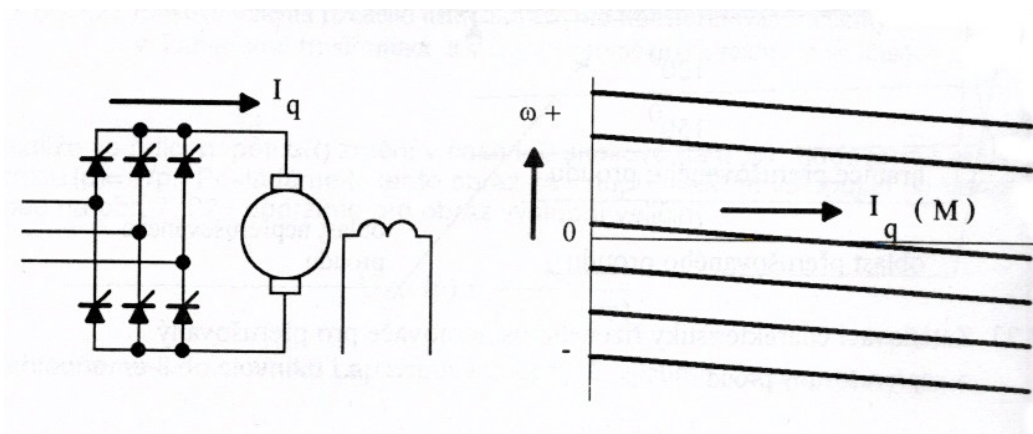
D.C. brush motor velocity control

- Control with resistance in the armature circuit



D.C. brush motor velocity control

- Speed velocity by the armature voltage



D.C. brush motor velocity control

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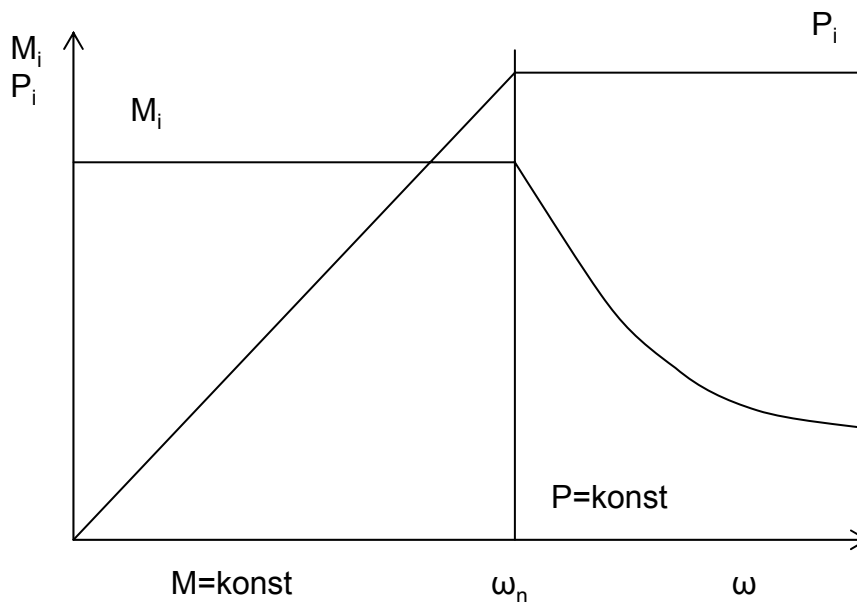
Model

Start up

Breaking

Control

- Speed velocity by excitation voltage
- (Beware of torque drop)





D.C. brush series motor

Introduction

Construction

Principle

Induced voltage

Torque

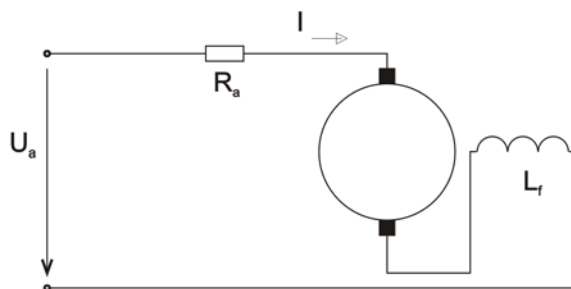
Model

Start up

Breaking

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Mathematical model



Unlinear system equation

$$u_a = R_a \cdot i_a + L_f \frac{di_a}{dt} + u_i$$

$$J \frac{d\omega}{dt} = M - M_z$$

$$u_i = k\Phi\omega = k \cdot c \cdot i_a \cdot \omega$$

$$M = k\Phi i_a = k \cdot c \cdot i_a^2$$

Static characteristics

$$1) \omega(i_a) \\ \text{if } \frac{d}{dt} = 0$$

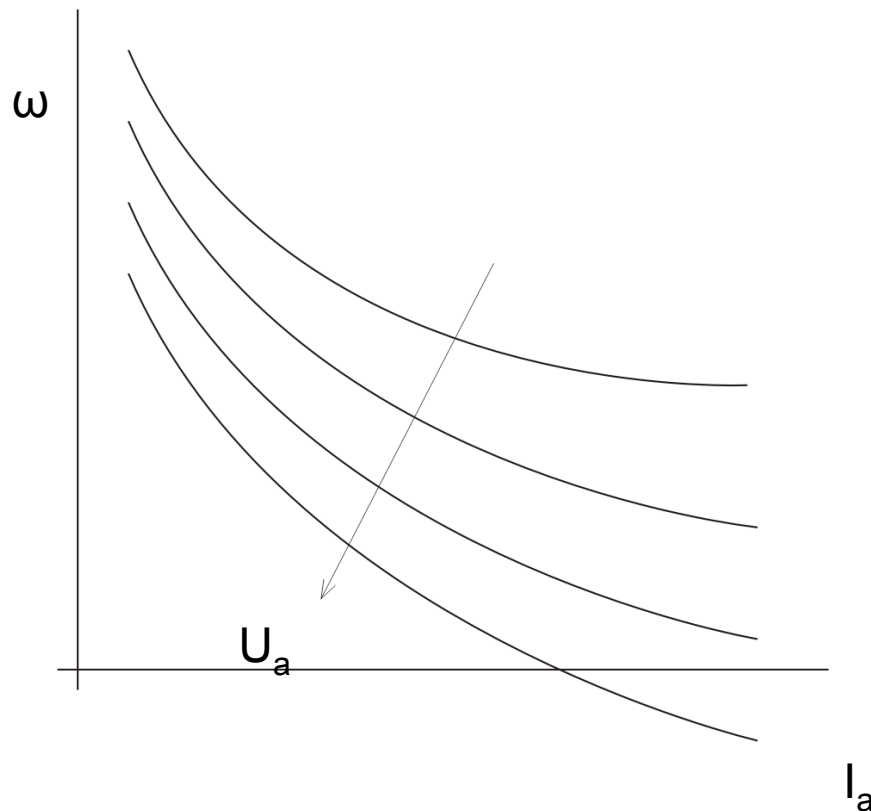
$$U_a = R_a \cdot I_a + k \cdot C \cdot I_a \cdot \omega$$

$$\Rightarrow \omega = \frac{U_a - R_a \cdot I_a}{k \cdot C \cdot I_a} = \frac{U_a}{k \cdot C \cdot I_a} - \frac{R_a}{k \cdot C}$$



D.C. brush series motor

Speed characteristic $\omega = f(I)$



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D.C. brush series motor

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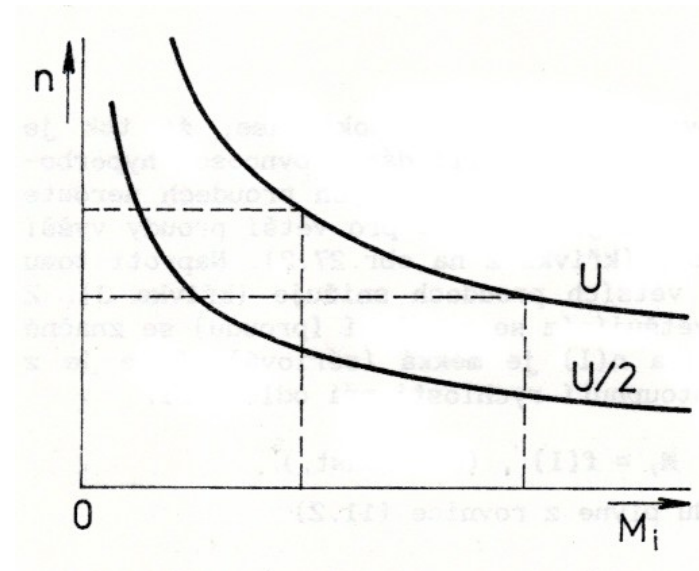
Breaking

Control

- Mechanical characteristic $n = f(M)$

$$M = k \cdot C \cdot I^2 \Rightarrow I_a = \sqrt{\frac{M}{kC}}$$

$$\omega = \frac{U_a}{k \cdot C \cdot \sqrt{\frac{M}{kC}}} - \frac{R_a}{kC} = \frac{U_a}{\sqrt{kC \cdot M}} - \frac{R_a}{kC}$$



D.C. brush series motor

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Torque

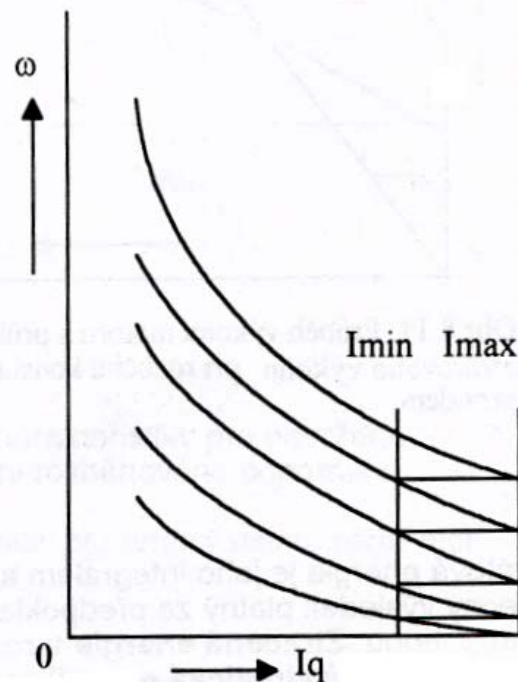
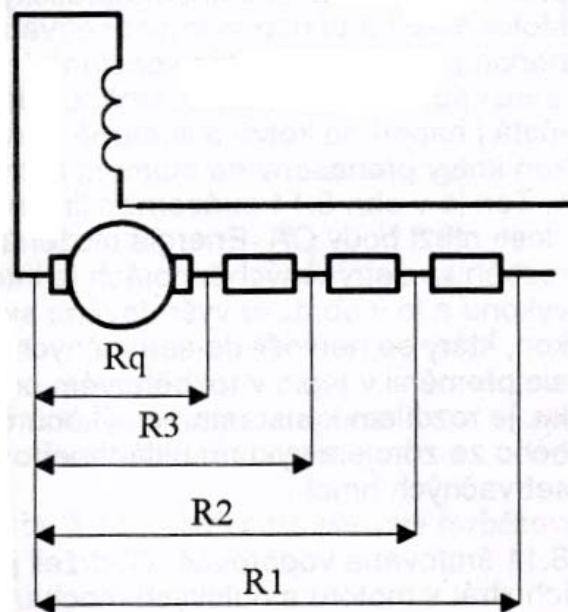
Model

Start up

Breaking

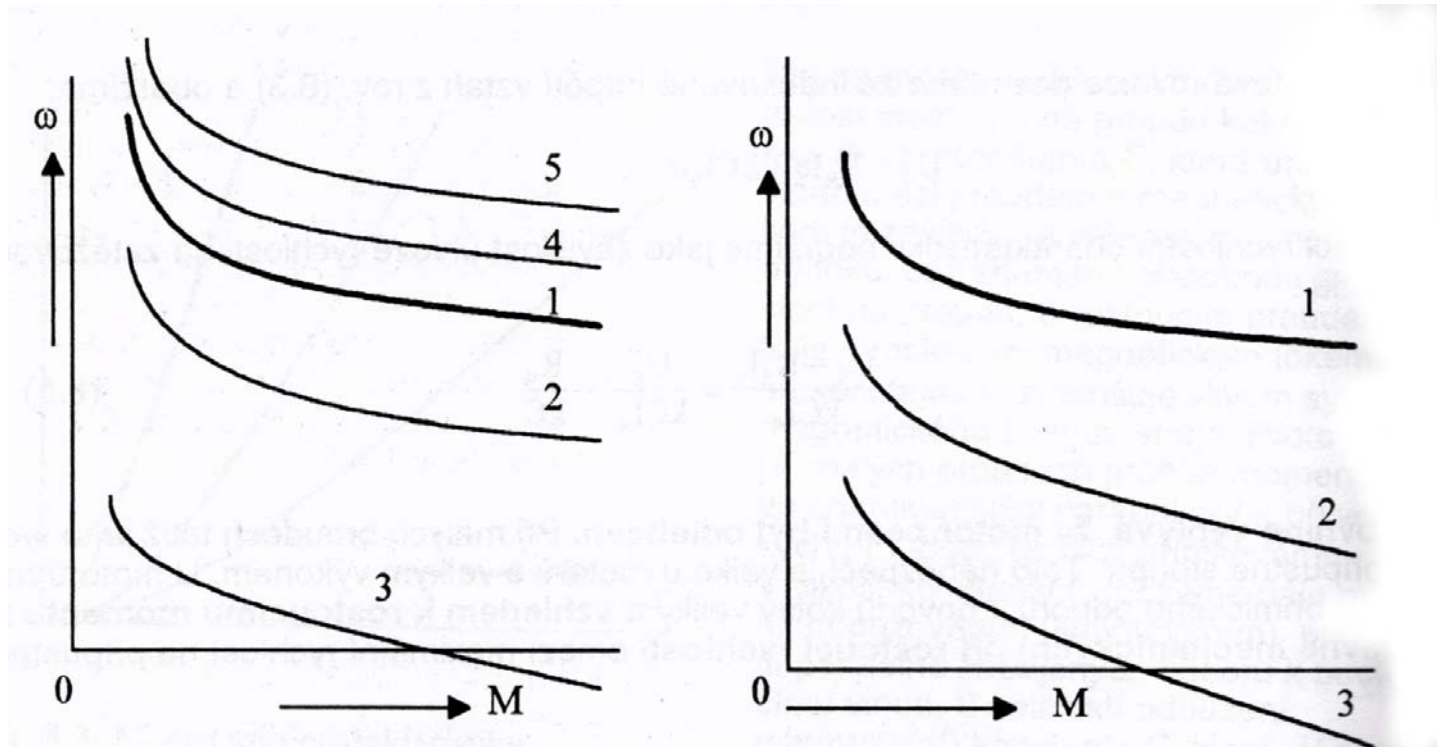
Control

- Anchor resistance starting



D.C. brush series motor

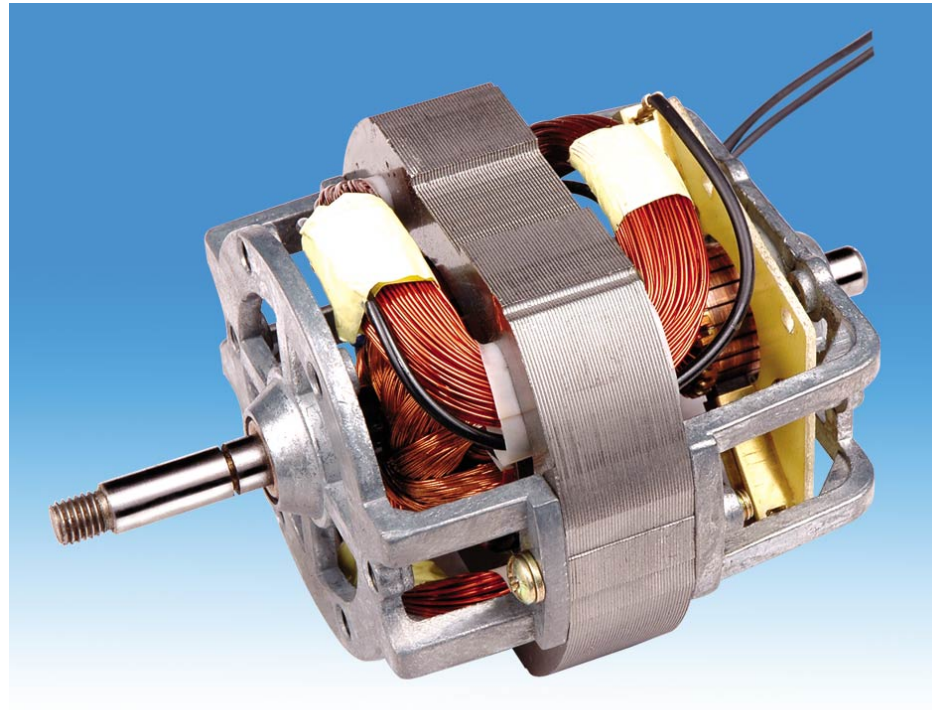
- Mechanical characteristics





D.C. brush series motor

- Recuperation's breaking
- A.C Brush series(universal) motor



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Thank you for your attention

Tento materiál vznikl v rámci projektu ESF CZ.1.07/2.2.00/28.0050
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